How I learnt to love typed λ-calculus

A NewCrafts 2025 Hands-On Lab

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Introduction

An experimental workshop to explore the computational foundation of the universe, and have fun

λ-Calculus Syntax

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- An *Application*: `(foo bar)`

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- β-reduction: `((lam (x) body) term)` $\Longrightarrow$ `body[x ← term]`

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- Local definitions: `(let ((id (lam (x) x))) (id foo))`
- Global definitions: `(def id (lam (x) x))`

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  - strings: `"λ-calculus rocks"`

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- There's a whole zoo of type systems we won't dive into (see you next year!)
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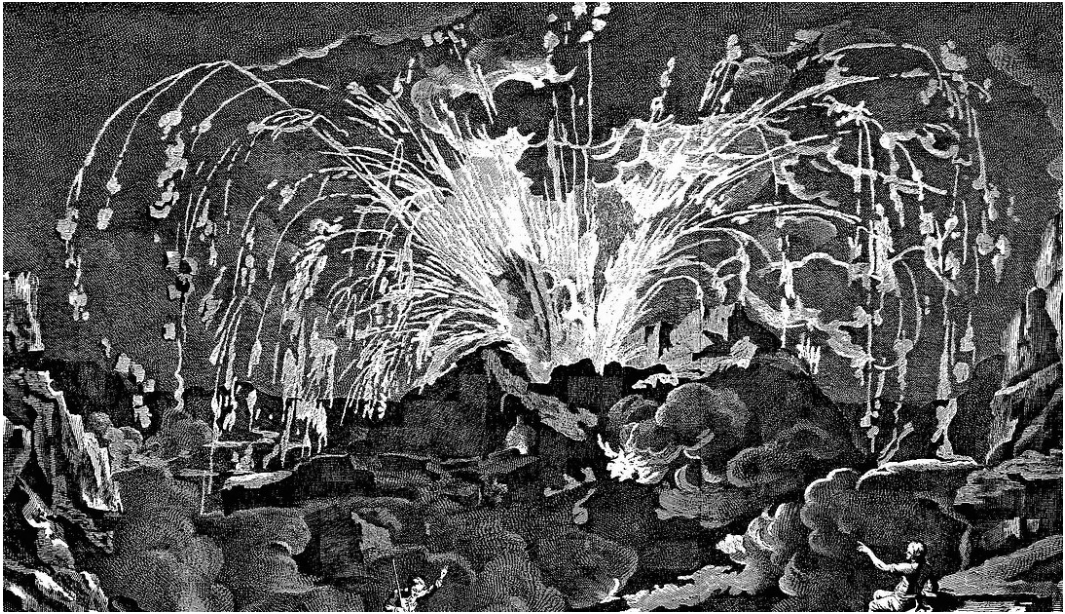
- We'll introduce *Simply Typed λ-Calculus* aka. $\lambda_{\rightarrow}$

Action



Conclusion